JEE Advanced 2026

Sample Paper - 2 (Paper-1)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +4 marks for the correct response and -1 for the incorrect response.

Physics

1. When a low flying aircraft passes over head, we sometimes notice a slight shaking of the picture on our TV screen. This is due to: **[3]**

a) vibration created by the passage of aircraft

b) interference of the direct signal received by the antenna with the weak signal reflected by the passing aircraft

c) diffraction of the signal received from the antenna

d) change of magnetic flux occurring due to the passage of aircraft

2. Two equal negative charges -q are fixed at points (0,- a) and (0, a) on y-axis. A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will **[3]**

a) Execute simple harmonic motion about the origin

b) Move to the origin and remain at rest

c) Execute oscillatory but not simple harmonic motion

d) Move to infinity

3. When X-rays pass through a strong uniform magnetic field, then they: **[3]**

a) get deflected in the direction perpendicular to the field

b) get deflected in the direction opposite to the field

c) do not get deflected at all

d) get deflected in the direction of the field

4. The orbit of a satellite moving around the earth in the equatorial plane, as viewed from the earth appears to be stationary. What is the radius (approximate) of the orbit? **[3]**

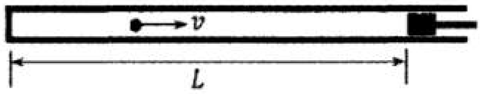
a) $\left(\frac{GM}{2\omega^2}\right)^{1/3}$

b) $\left(\frac{GM}{\omega^2}\right)^{1/2}$

c) $\left(\frac{GM}{\omega^2}\right)^{1/3}$

d) $\left(\frac{2GM}{\omega^2}\right)^{1/3}$

5. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low-speed V such that $V \lll \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. **[4]**



a) The rate at which the particle strikes the piston is v/L

b) After each collision with the piston, the particle speed increases by 2V.

c) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

d) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

6. The binding energy of nucleons in a nucleus can be affected by the pairwise Coulomb repulsion. Assume that all nucleons are uniformly distributed inside the nucleus. **[4]**

Let the binding energy of a proton be E_b^p and the binding energy of a neutron be E_b^n in the nucleus.

Which of the following statement(s) is(are) correct?

a) $E_b^p - E_b^n$ is positive.

b) E_b^p increases if the nucleus undergoes a beta decay emitting a positron

c) $E_b^p - E_b^n$ is proportional to Z(Z - 1) where Z is the atomic number of the nucleus.

d) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

7. A series R - C circuit is connected to AC voltage source. Consider two cases: **[4]**

A. when C is without a dielectric medium and

B. when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_c across the capacitor are compared in the two cases. Which of the following is/are true?

a) $V_C^A > V_C^B$

b) $V_C^A < V_C^B$

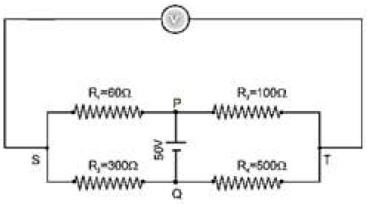
c) $I_R^A < I_R^B$

d) $I_R^A > I_R^B$

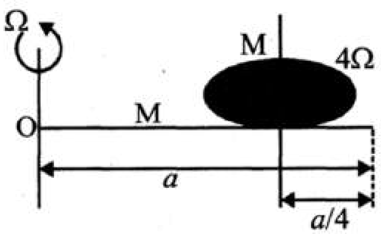
8. If the ionisation potential of hydrogen atom is 13.6 eV, the energy required to remove the electron from the third orbit of hydrogen atom is _________ eV. **[4]**

9. In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient $0.0004°C^{-1}$. If the temperature of R_3 is increased by 100°C, **[4]**

the voltage developed between S and T will be ________ volt.



10. A thin rod of mass M and length α is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius a/4 is pivoted on this rod with its center at a distance a/4 from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)$n. The value of n is ________. **[4]**



11. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10μ, then N is **[4]**

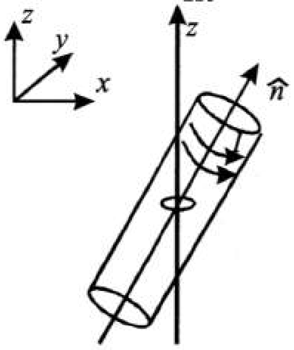
12. The specific heat capacity of a substance is temperature dependent and is given by the formula C = kT, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from –73 ^{o}C to 27 ^{o}C is nk. the value of n is [Given: 0 K = –273 ^{o}C.] **[4]**

13. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes the closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is [Take the proton mass, $m_p = \left(\frac{5}{3}\right) \times 10^{-27}$ kg; $\frac{h}{e} = 4.2 \times 10^{-15}$ J-s/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ m/F; 1 fm = 10^{-15} m] **[4]**

14. A small circular loop of area A and resistance R is fixed on a horizontal xy-plane with the center of the loop always on the axis $\hat{n}$ of a long solenoid. The solenoid has m turns per unit length and carries current I counterclockwise as shown in the **[4]**

figure. The magnetic field due to the solenoid is in $\hat{n}$ direction. List - I gives time dependences of in terms of a constant angular frequency ω List - II gives the torques experienced by the circular loop at time $t = \frac{\pi}{6\omega}$

Let $\alpha = \frac{A^2\mu_0^2 m^2 I^2 \omega}{2R}$



List - I	**List - II**
(I) $\frac{1}{\sqrt{2}}(\sin\omega t\hat{j} + \cos\omega t\hat{k})$	(P) 0
(II) $\frac{1}{\sqrt{2}}(\sin\omega t\hat{i} + \cos\omega t\hat{j})$	(Q) $-\frac{\alpha}{4}\hat{i}$
(III) $\frac{1}{\sqrt{2}}(\sin\omega t\hat{i} + \cos\omega t\hat{k})$	(R) $\frac{3\alpha}{4}\hat{i}$
(IV) $\frac{1}{\sqrt{2}}(\cos\omega t\hat{j} + \sin\omega t\hat{k})$	(S) $\frac{\alpha}{4}\hat{j}$
	(T) $-\frac{3\alpha}{4}\hat{i}$

Which one of the following options is correct?

a) (I) $\to$ (Q); (II) $\to$ (P); (III) $\to$ (S); (IV) $\to$ (R)

b) (I) $\to$ (T); (II) $\to$ (Q); (III) $\to$ (P); (IV) $\to$ (R)

c) (I) $\to$ (Q); (II) $\to$ (P); (III) $\to$ (S); (IV) $\to$ (T)

d) (I) $\to$ (S); (II) $\to$ (T); (III) $\to$ (Q); (IV) $\to$ (P)

15. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. **[4]**

Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists.

List - I	**List - II**
(P) Lift is accelerating vertically up	(1) d = 1.2 m

List - I	List - II
(Q) Lift is accelerating vertically down with an acceleration less than the gravitational acceleration	(2) d > 1.2 m
(R) Lift is moving vertically up with constant speed	(3) d < 1.2 m
(S) Lift is falling freely	(4) No water leaks out of the jar

a) P - 1, Q - 1, R - 1, S - 4

b) P - 2, Q - 3, R - 1, S - 4

c) P - 2, Q - 3, R - 2, S - 4

d) P - 2, Q - 3, R - 1, S - 1

16. A musical instrument is made using four different metal strings 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0. List - I gives the above four strings while list - II lists the magnitude of some quantity. **[4]**

List-I	List-II
(I) String - 1 (μ)	(P) 1
(II) String - 2 (2μ)	(Q) $\frac{1}{2}$
(III) String - 3 (3μ)	(R) $\frac{1}{\sqrt{2}}$
(IV) String - 4 (4μ)	(S) $\frac{1}{\sqrt{3}}$
	(T) $\frac{3}{16}$
	(U) $\frac{1}{16}$

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

a) (I) $\rightarrow$ (Q), (II)$\rightarrow$ (S), (III)$\rightarrow$ (R), (IV)$\rightarrow$ (P)

b) (I) $\rightarrow$ (Q), (II)$\rightarrow$ (P), (III)$\rightarrow$ (R), (IV)$\rightarrow$ (T)

c) (I) $\rightarrow$ (P), (II)$\rightarrow$ (Q), (III)$\rightarrow$ (T), (IV)$\rightarrow$ (S)

d) (I) $\rightarrow$ (P), (II)$\rightarrow$ (R), (III)$\rightarrow$ (S), (IV)$\rightarrow$ (Q)

Chemistry

17. The number of moles of $KMnO_4$ that will be needed to react with one mole of sulphite ion in acidic solution is [3]

a) $\frac{3}{5}$

b) 1

c) $\frac{4}{5}$

d) $\frac{2}{5}$

18. Which of the following fcc structure contains cations in alternate tetrahedral voids? [3]

a) Na_2O

b) ZnS

c) CaF_2

d) NaCl

19. When the temperature is increased, surface tension of water [3]

a) remains constant

b) increases

c) shows irregular behaviour

d) decreases

20. $^{27}_{13}Al$ s a stable isotope, $^{29}_{13}Al$ is expected to disintegrate by [3]

a) proton emission

b) positron emission

c) α-emission

d) β-emission

21. For the reaction [4]

$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$

The correct statement(s) in the balanced equation is/are

a) H_2O is one of the products

b) Sulphur is reduced

c) Stoichiometric coefficient of HSO_4^- is 6

d) Iodide is oxidized

22. Each of the following options contains a set of four molecules. Identify the option(S) were all four molecules posses permanent dipole moment at room temperature. [4]

a) SO_2, C_6H_5Cl, H_2Se, BrF_5

b) $BeCl_2$, CO_2, BCl_3, $CHCl_3$

c) NO_2, NH_3, $POCl_3$, CH_3Cl

d) BF_3, O_3, SF_6, XeF_6

23. Addition of high proportions of manganese makes steel useful in making rails of railroads, because manganese [4]

a) can remove oxygen and sulphur

b) can show the highest oxidation state of +7.

c) gives hardness to steel

d) helps the formation of oxides of iron

24. A trinitro compound, 1, 3, 5-tris-(4-nitrophenyl)benzene, on complete reaction with an excess of $\frac{Sn}{HCl}$ gives a major product, which on treatment with an excess of $\frac{NaNO_2}{HCl}$ at 0 °C provides P as the product. P, upon treatment with excess of H_2O at room temperature, gives the product Q. Bromination of Q in aqueous medium furnishes the product R. The compound P upon treatment with an excess of phenol under basic conditions gives the product S. The molar mass difference between compounds Q and R is 474 g mol^{-1} and between compounds P and S is 172.5 g mol^{-1}. **[4]**

The number of heteroatoms present in one molecule of R is ________.

[**Use:** Molar mass (in $gmol^{-1}$): H = 1, C = 12, N = 14, O = 16, Br = 80, Cl = 35.5

Atoms other than C and H are considered as heteroatoms]

25. Among the following, the number of elements showing only one non-zero oxidation state is: **[4]**

O, Cl, F, N, P, Sn, Tl, Na, Ti

26. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas Q. The amount of $CuSO_4$ (in g) required to completely consume the gas Q is ________. **[4]**

[**Given:** Atomic mass of H = 1, O = 16, Na = 23, P = 31, S = 32, Cu = 63]

27. In an estimation of bromine by Carius method, 1.6 g of an organic compound gave 1.88 g of AgBr. The mass percentage of bromine in the compound is ________. **[4]**

(Atomic mass, Ag = 108, Br = 80 g mol^{-1})

28. A closed tank has two compartments A and B, both filled with oxygen (assumed to be ideal gas). The partition separating the two compartments is fixed and is a perfect heat insulator (Fig. 1). If the old partition is replaced by a new partition that can slide and conduct heat but does not allow the gas to leak across (Fig. 2), the **[4]**

volume (in m^3) of compartment A after the system attains equilibrium is ________.

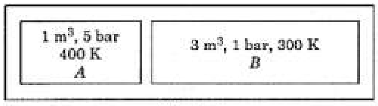
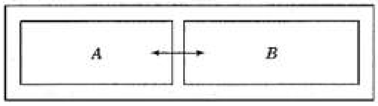


Fig. 1



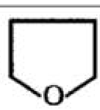
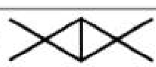
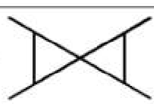
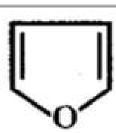
29. The total number of cyclic structural, as well as stereoisomers possible for a compound with the molecular formula C_5H_{10}, is: **[4]**

30. Match List-I with List-II. **[4]**

List-I	**List-II**
(A) Antipyretic	(I) Reduces pain
(B) Analgesic	(II) Reduces stress
(C) Tranquilizer	(III) Reduces fever
(D) Antacid	(IV) Reduces acidity (Stomach)

a) A - I, B - III, C - II, D - IV
b) A - III, B - I, C - IV, D - II
c) A - III, B - I, C - II, D - IV
d) A - I, B - IV, C - II, D - III

31. Match List-I with List-II. **[4]**

List-I	**List-II**
(A) 	(I) Spiro compound
(B) 	(II) Aromatic compound
(C) 	(III) Non-planar Heterocyclic compound
(D) 	(IV) Bicyclo compound

a) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)
b) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)
c) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)
d) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

32. Match List-I with List-II **[4]**

(List-I)	(List-II)
Natural amino acid	**One letter code**
(A) Glutamic acid	(I) Q
(B) Glutamine	(II) W
(C) Tyrosine	(III) E
(D) Tryptophan	(IV) Y

a) A - III, B - I, C - IV, D - II
b) A - IV, B - III, C - I, D - II
c) A - III, B - IV, C - I, D - II
d) A - II, B - I, C - IV, D - III

Maths

33. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN, is [3]

a) 360
b) 96
c) 48
d) 192

34. If the events A and B are mutually exclusive events such that P(A) = $\frac{3x+1}{3}$, P(B) = $\frac{1-x}{4}$, then the set of possible values of x lies in the interval: [3]

a) $\left[-\frac{1}{3}, \frac{5}{9}\right]$
b) $\left[-\frac{7}{9}, \frac{4}{9}\right]$
c) [0, 1]
d) $\left[\frac{1}{3}, \frac{2}{3}\right]$

35. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is creacted at the point O such ahat $\angle PAO = \angle PBO = 15^\circ$ and $PCO = 45^\circ$ where C is the midpoint of AB. Then $(OP)^2$ is equal to [3]

a) $\frac{32}{\sqrt{3}}(2-\sqrt{3})$
b) $\frac{32}{\sqrt{3}}(\sqrt{3}-1)$
c) $\frac{16}{\sqrt{3}}(2-\sqrt{3})$
d) $\frac{16}{\sqrt{3}}(\sqrt{3}-1)$

36. The order of the differential equation whose general solution is given by y = (c_1 + c_2) cos (x + c_3) - $c_4e^{x+c_5}$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is [3]

a) 5
b) 3

c) 2

d) 4

37. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \ldots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \geq 1$. Then, which of the following is/are TRUE? **[4]**

a) $\sum_{k=1}^{30} T_k = 35610$

b) $T_{30} = 3454$

c) $\sum_{k=1}^{20} T_k = 10510$

d) $T_{20} = 1604$

38. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1? **[4]**

a) $x - y = 0$

b) $x + y = 0$

c) $x + 7y = 0$

d) $x - 7y = 0$

39. If $x + |y| = 2y$, then y as a function of x is **[4]**

a) differentiable for all x

b) continuous at $x = 0$

c) defined for all real x

d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

40. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is ________. **[4]**

41. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where $|A|$ denotes the determinant of A. Then the number of elements in S is ________. **[4]**

42. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then the value of $\vec{r} \cdot \vec{b}$ is: **[4]**

43. For $x \in R$, let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the minimum value of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \int_0^{x \tan^{-1} x} \frac{e^{(t - \cos t)}}{1 + t^{2023}} dt$ is **[4]**

44. The maximum value of the function f(x) = $2x^3 - 15x^2 + 36x - 48$ on the set A = {x | $x^2 + 20 \leq 9x$} is **[4]**

45. Let $\bar{z}$ denote the complex conjugate of a complex number z and let i = $\sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation $\bar{z} - z^2 = i\left(\bar{z} + z^2\right)$ is ________. **[4]**

46. Two players, P_1 and P_2, play a game against each other. In every round of the game, each player rolls a fair die once, where the six faces of the die have six distinct numbers. Let x and y denote the readings on the die rolled by P_1 and P_2 respectively. If x > y, then P_1 scores 5 points and P_2 scores 0 point. If x = y, then each player scores 2 points. If x < y, then P_1 scores 0 point and P_2 scores 5 points. Let X_i and Y_i be the total scores of P_1 and P_2 respectively, after playing the i^{th}Ground. **[4]**

List-I	List-II
(I) Probability of $(X_2 \geq Y_2)$ is	(P) $\frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	(Q) $\frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	(R) $\frac{5}{16}$
(IV) Probability of $(X_3 = Y_3)$ is	(S) $\frac{355}{864}$
	(T) $\frac{77}{432}$

a) (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)

b) (I) → (P); (II) → (R); (III) → (Q); (IV) → (S)

c) (I) → (Q); (II) → (R); (III) → (T); (IV) →(S)

d) (I) → (Q); (II) → (R); (III) → (T); (IV) → (T)

47. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and and $j = 1, 2, 3$. **[4]**
Match each entry in List-I to the correct entry in List-II.

List-I		List-II	
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12

(R)	Let $M=(a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij}\in T$ for $i>j$. Then the number of elements in the set $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}: x,y,z\in\mathbb{R}, M\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}\right\}$ is	(3)	infinite
(S)	Let $M=(a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i=0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

a) $(P)\to(2),(Q)\to(4),(R)\to(1),(S)\to(5)$

b) $(P)\to(2),(Q)\to(4),(R)\to(3),(S)\to(5)$

c) $(P)\to(1),(Q)\to(5),(R)\to(3),(S)\to(4)$

d) $(P)\to(4),(Q)\to(2),(R)\to(5),(S)\to(1)$

48. Consider the lines $L_1:\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}$, $L_2\frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes P_1 : 7x + y + 2z = 3, P_2 = 3x + 5y - 6 z = 4. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2. **[4]**

Match List I with List II and select the correct answer using the code given below the lists :

List I	**List II**
P. $a=$	1. 13
Q. $b=$	2. -3
R. $c=$	3. 1
S. $d=$	4. -2

a) (P) → (3), (Q) → (2), (R) → (4), (S) → (1)

b) (P) → (2), (Q) → (4), (R) → (1), (S) → (3)

c) (P) → (3), (Q) → (2), (R) → (1), (S) → (4)

d) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)

Solution

Physics

1.

(b) interference of the direct signal received by the antenna with the weak signal reflected by the passing aircraft

Explanation:

interference of the direct signal received by the antenna with the weak signal reflected by the passing aircraft

2.

(c) Execute oscillatory but not simple harmonic motion

Explanation:

Motion is simple harmonic only if Q is released from a point not very far from the origin on x-axis. Otherwise motion is periodic but not simple harmonic.

3.

(c) do not get deflected at all

Explanation:

Because X-rays are electromagnetic (Neutral) in nature.

4.

(c) $(\frac{GM}{\omega^2})^{1/3}$

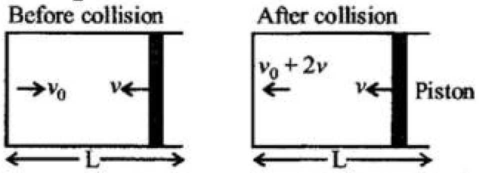
Explanation:

$T^2 = \frac{4\pi^2}{GM}a^3$ or $\left(\frac{2\pi}{\omega}\right)^2 = \frac{4\pi^2}{GM}a^3$ as $T = \frac{2\pi}{\omega}$

$\frac{4\pi^2}{\omega^2} = \frac{4\pi^2}{GM}a^3$ or $a^3 = \left(\frac{GM}{\omega^2}\right)$ $a = (\frac{GM}{\omega^2})^{1/3}$

5. **(b)** After each collision with the piston, the particle speed increases by 2V.

(c) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

Explanation: 

When the small particle moving with velocity v_0 undergoes an elastic collision with the heavy movable piston moving with velocity v, it acquires a new velocity $v_0 + 2v$. So, the increase in velocity after every collision is 2v.

Time period of collision when the piston is at a distance 'L' from the closed end

$T = \frac{\text{distance}}{\text{speed}} = \frac{2L}{v'}$

Where v' is the speed of the particle at that time.

$\therefore$ Frequency or rate at which the particle strikes the piston

$= \frac{v'}{2L}$

The rate of change of speed of the particle

$= \frac{dv'}{dt} = (\text{ frequency}) \times 2v \therefore dv' = \frac{v'}{2L} 2vdt$

$\therefore \frac{dv'}{v'} = \frac{vdt}{L} = \frac{-dL}{L}$

Where dL is the distance travelled by the piston in time dt.

The minus sign indicates decrease in 'L' with time.

$\therefore \int_{V_0}^{v'} \frac{dv'}{v'} = -\int_{L_0}^{x} \frac{dL}{L}$

$\therefore \ln \frac{v'}{v_0} = -\ln \frac{L}{L_0}$ or $|v'| = \frac{v_0 L_0}{L}$

When $L = \frac{L_0}{2}$ we have $|V'| = \frac{v_0 L_0}{L_0/2} = 2V_0$

$\therefore K \cdot E_{L_{0/2}} = \frac{1}{2} m(2v_0)^2$

$\therefore K \cdot E_{L_0} = \frac{1}{2} mv_0^2 \therefore \frac{K \cdot E_{L_{0/2}}}{K \cdot E_{L_0}} = 4$

6. **(b)** E_b^p increases if the nucleus undergoes a beta decay emitting a positron

(c) $E_b^p - E_b^n$ is proportional to Z(Z - 1) where Z is the atomic number of the nucleus.

(d) $E_b^p - E_b^n$ is proportional to $A^{-\frac{1}{3}}$ where A is the mass number of the nucleus.

Explanation: Binding energy of proton and neutron due to nuclear force is same. So difference is due to electrostatic potential repulsion energy and it is + ve. So $E_b^p - E_b^n =$ electrostatic potential energy.

Number of proton pair $= Z_{C_2} = \frac{Z(Z-1)}{2}$

So, repulsion energy $\propto \frac{Z(Z-1)}{2} \times \frac{1}{4\pi\epsilon_0} \frac{e^2}{R}$,

R = radius of nuclei

$E_b^p - E_b^n \propto Z(Z-1)$ is correct.

As $R = R_0 A^{1/3}$

So, $E_b^p - E_b^n \propto A^{-1/3}$ is correct.

Because of repulsion

$E_b^p < E_b^n$ is incorrect.

Since in β^+decay, number of proton decrease $\Rightarrow$ repulsions decreases $\Rightarrow E_b^p$ increases is correct.

7. **(a)** $V_C^A > V_C^B$

(d) $I_R^A > I_R^B$

Explanation: In RC - circuit impedance, $Z = \sqrt{R^2 + \left(\frac{1}{WC}\right)^2}$

The capacitance in case B is four times the capacitance in case A

$\therefore$ Impedance in case B is less then that of case A($Z_B < Z_A$)

Now I = $\frac{V}{Z}$ $\therefore I_R^A < I_R^B$. and $V_R^A < V_R^B$. $\Rightarrow V_C^A > V_C^B$

[$\because$ If V is the applied potential difference access series R-C circuit then $V = \sqrt{V_R^2 + V_C^2}$]

8. 1.5

Explanation:

Energy required to remove the electron from the third orbit

$= \frac{13.6}{3^2}\text{eV} = 1.5\text{eV} = 1.5 \times 1.6 \times 10^{-19}\,\text{J}$

$= 2.4 \times 10^{-19}$ J = 1.5 eV.

9. 0.27

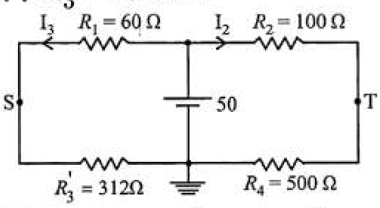
Explanation:

According to question, resistance R_3 has temperature coefficient, $\alpha = 0.0004°\text{C}^{-1}$

And temperature is increased by 100°C i.e., $\Delta T = 100°$ C

$R'_3 = R_0(1 + \alpha\Delta T) = 300(1 + \alpha\Delta T)$

$\therefore R'_3 = 312\Omega$



Here $I_1 = \frac{V}{R_1+R_3} = \frac{50}{372}$ and $I_2 = \frac{V}{R_2+R_4} = \frac{50}{600}$

$V_S - V_T = 312I_1 - 500I_2$

$= 312 \times \frac{50}{372} - 50 \times \frac{500}{600} = 41.94 - 41.67 = 0.27$ V

Hence voltage developed between S and T = 0.27 Volt

10. 49

Explanation:

Angular momentum of the system, $L_S = L_{rod} + L_{disc}$

$= I_{rod}\omega + I_{disc}\omega + \vec{r} \times \vec{P}$

$= \frac{Ma^2}{12}\omega + \frac{Ma^2\omega}{32} + \frac{3a}{16} \times \frac{3a}{4} \times M \times \omega$

$= \frac{Ma^2}{12} \times 4\Omega + \frac{Ma^2}{32} \times 4\Omega + \frac{3a}{16} \times \frac{3a}{4} \times M \times 4\Omega$

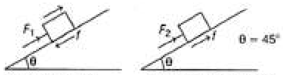
$\therefore L_s = \frac{Ma^2}{3}\Omega + M\left(\frac{3a}{4}\right)^2\Omega \frac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$

or $L_s = \frac{49}{48}Ma^2\Omega$

$\therefore$ n = 49

11. 5

Explanation:



Moving upwards Just remains stationary

$F_1 = mg \sin\theta + \mu mg \cos\theta$

$F_2 = mg \sin\theta - \mu mg \cos\theta$

Given that $F_1 = 3F_2$ or $(\sin 45^o + \mu\cos 45^o)$

$= 3(\sin 45^o - \mu \cos 45^o)$

On solving, we get $\mu = 0.5$

$\therefore N = 10\mu = 5$

12. 25000.0

Explanation:

Specific heat, $C = \frac{dQ}{mdT}$

$dQ = mCdT$

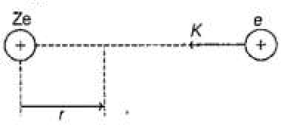
$dQ = kTdT[\text{ As } m = 1\ kg]$

$Q = \int_{200}^{300} kTdT = k\left[\frac{T^2}{2}\right]_{200}^{300} = \frac{k}{2}\left[300^2 - 200^2\right]$

$Q = 25000k$

13. 7

Explanation:



r = closest distance = 10 fm.

From energy conservation, we have

$K_i + U_i = K_f + U_f$

or $K + 0 = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

or $K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(120e)(e)}{r}$ ...(i)

de-Broglie wavelength

$\lambda = \frac{h}{\sqrt{2Km}}$ ...(ii)

Substituting the given values in above two equations, we get

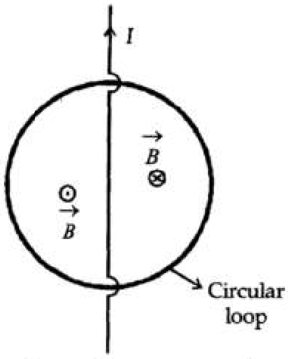
$\lambda = 7 \times 10^{-15}$ m = 7 fm

14. **(a)** (I) $\rightarrow$ (Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (R)

Explanation:

If the current is constant, the emf induced in the loop zero. Emf will be induced in the circular wire loop when flux through it changes with time.

$e = -\frac{\Delta\phi}{\Delta t}$



when the current is constant, the flux changing through it will be zero.

Also, if the current decreases at steady rate, the emf. induced in the loop is zero.

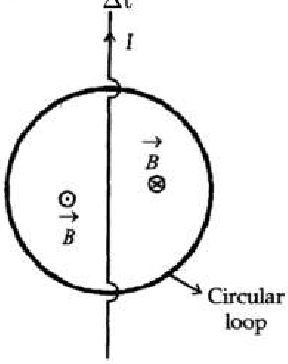
When the current is decreasing at a steady rate then the change in the flux (decreasing inwards) on the right half of the wire is equal to the change in flux (decreasing outwards) on the left half of the wire such that $\Delta\phi$ through the circular loop is zero.

(c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (P); (III) $\rightarrow$ (S); (IV) $\rightarrow$ (T)

Explanation:

If the current is constant, the emf induced in the loop zero. Emf will be induced in the circular wire loop when flux through it changes with time.

$e = -\frac{\Delta\phi}{\Delta t}$



when the current is constant, the flux changing through it will be zero.

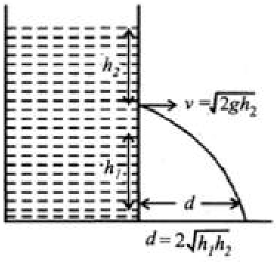
Also, if the current decreases at steady rate, the emf. induced in the loop is zero.

When the current is decreasing at a steady rate then the change in the flux (decreasing inwards) on the right half of the wire is equal to the change in flux (decreasing outwards) on the left half of the wire such that $\Delta\phi$ through the circular loop is zero.

15. **(a)** P - 1, Q - 1,R - 1, S - 4

Explanation:

Horizontal distance,



$d = y \times t$

$= \sqrt{2gh_2} \times \sqrt{\frac{2h_1}{g}}$

$= 2\sqrt{h_1 h_2}$

If $g_{eff} > g$

$g_{eff} = g$

$g_{eff} < g$

In all the three cases $d = 2\sqrt{h_1 h_2} = 1.2$ m

If $g_{eff} = 0$, then no water leaks out as there will be no pressure difference.

16.

(d) (I) $\rightarrow$ (P), (II)$\rightarrow$ (R), (III)$\rightarrow$ (S), (IV)$\rightarrow$ (Q)

Explanation:

Frequency, $v = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ for first mode of vibration

For 'v' to be maximum, 'l' should be minimum.

String - 1 $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

String - 2 $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}}$

String - 3 $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{\sqrt{3}}$

String - 4 $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

Chemistry

17.

(d) $\frac{2}{5}$

Explanation:

The reaction of MnO_4^- and SO_3^{2-} in acidic medium is derived as follows:

$$
\begin{array}{r}
MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O] \times 2 \\
SO_3^{2-} + H_2O \rightarrow SO_4^{2-} + 2H^+ + 2e^-] \times 5 \\
\hline
2MnO_4^- + 5SO_3^{2-} + 6H^+ \rightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O
\end{array}
$$

Hence, 2 mole $2MnO_4^- \equiv 5$ mol SO_3^{2-}

i.e., $\frac{2}{5}$ mol $MnO_4^- \equiv 1$ mol SO_3^{2-}

18.

(b) ZnS

Explanation:

In ZnS, S^{2-} (sulphide ions) are present at fee positions giving our sulphide ions per unit cell. To comply with 1 : 1 stoichiometry, four Zn^{2+} ions must be present in four alternate tetrahedral voids out of eight tetrahedral voids present.

In NaCl, Na^+ ions are present in octahedral voids while in Na_2O, Na^+ ions are present in all its tetrahedral voids giving the desired 2 : 1 stoichiometry. In CaF_2, Ca^{2+} ions occupies fee positions and all the tetrahedral voids are occupied by fluoride ions.

19.

(d) decreases

Explanation:

As temperature increases surface tension of liquid decreases.

20.

(d) β-emission

Explanation:

The species ${}_{13}Al^{29}$ (No. of neutrons = 16) contains more neutrons than the stable isotope ${}_{13}Al^{27}$ (No. of neutrons = 14).

Neutron on decomposition shows β-emission.

$${}_0n^1 \rightarrow {}_{+1}p^1 + \underset{\beta-particle}{{}_{-1}e^0}$$

21. **(a)** H_2O is one of the products

(c) Stoichiometric coefficient of HSO_4^- is 6

(d) Iodide is oxidized

Explanation: Balancing the chemical equation by half-reaction method.

$$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$$

$$2I^- \longrightarrow I_2 + 2e^- \times 2e^- \times 3$$

$$\frac{ClO_3^- + 6H^+ + 6e^- \longrightarrow Cl^- + 3H_2O}{ClO_3^- + 6I^- + 6H^+ \longrightarrow 3I_2 + Cl^- + 3H_2O}$$

Adding $6HSO_4^-$ to both sides.

$$ClO_3^- + 6I^- + 6H_2SO_4 \longrightarrow 3I_2 + Cl^- + 6HSO_4^- + 3H_2O$$

22. **(a)** SO_2, C_6H_5Cl, H_2Se, BrF_5

(c) NO_2, NH_3, $POCl_3$, CH_3Cl

Explanation: Dipole moment (μ) value of BF_3, SF_6, $BeCl_2$, CO_2, BCl_3 is zero.

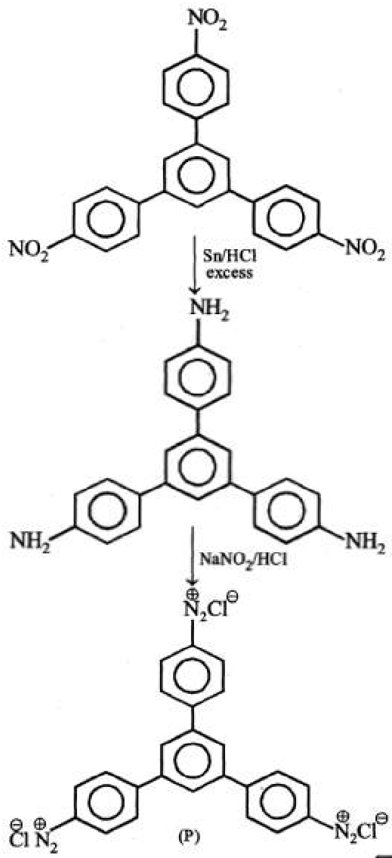
23. **(a)** can remove oxygen and sulphur

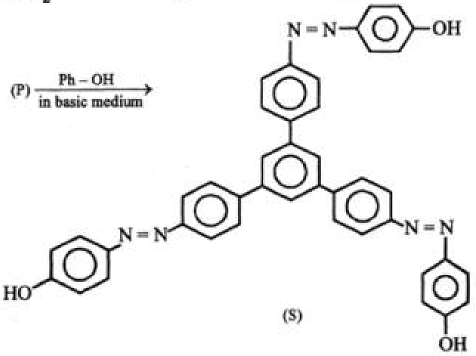
(c) gives hardness to steel

Explanation: Mn makes steel harder and increases its elasticity and tensile strength. Further Mn acts as deoxidiser. MnO reacts with S present in cast iron, gets oxidised and then combine to form slag.

24. 9.0

Explanation:

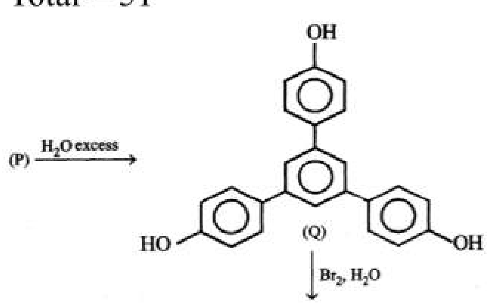


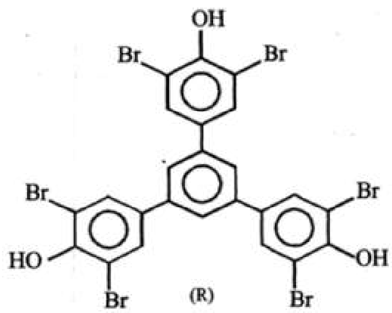


Number of carbon atoms = 42

Number of hetero atoms = 09

Total = 51





Number of hetero atoms in R is 9.

25. 2

Explanation:

Fluorine generally shows 0 and -1 oxidation states while sodium shows 0 and +1 oxidation state.

26. 2.38

Explanation:

$$\underset{\text{(White phosphorous)}}{P_4} + 3NaOH + 3H_2O \xrightarrow[\text{atmosphere}]{\text{Inert}} \underset{\text{Phosphine}}{PH_3} \uparrow + 3NaH_2PO_2$$

PH_3 : a non-inflammable gas in its pure form; slightly soluble in water.

When PH_3 is absorbed in $CuSO_4$ solution cupric phosphide forms:

$2PH_3 + 3CuSO_4 \longrightarrow CuP_2 + 3H_2SO_4$

1 mol of $P_4 = 31 \times 4 = 124$g

$\therefore$ 1.24 g of white phosphorous = 0.01 mol

$\therefore$ 0.01 mol of P_4 forms 0.01 mol of P_3

No. of moles of $CuSO_4$ is required for complete consumption of 0.01 mol = $0.01 \times \frac{3}{2} = 15 \times 10^{-3}$

M. W. of $CuSO_4$ = 159 g/mol

$\therefore$ Amount of $CuSO_4$ required = $15 \times 10^{-3} \times 159 = 2.38$ g

27. 50.0

Explanation:

$$\% \text{ of Br} = \frac{\text{Wt. of AgBr}}{\text{Wt. of O.C.}} \times \frac{\text{Molar mass of Br}}{\text{Molar mass of AgBr}} \times 100$$

$= \frac{1.88}{1.6} \times \frac{80}{188} \times 100 = 50\%$

28. 2.22

Explanation:

Given $p_1 = 5$ bar, $V_1 = 1m^3$, $T_1 = 400K$

So, $n_1 = \frac{5}{400R}$ (from pV = nRT)

Similarly, $p_2 = 1$ bar, $V_2 = 3\ m^3$, $T_2 = 300K$, $n_2 = \frac{3}{300R}$

Let at equilibrium the new volume of A will be (1 + x)

So, the new volume of B will be (3 - x)

Now, from the ideal gas equation.

$\frac{p_1 V_1}{n_1 R T_1} = \frac{p_2 V_2}{n_2 R T_2}$

and at equilibrium (due to conduction of heat)

$\frac{p_1}{T_1} = \frac{p_2}{T_2}$

So, $\frac{V_1}{n_1} = \frac{V_2}{n_2}$ or $V_1 n_2 = V_2 n_1$

After putting the values

$(1+x) \times \frac{3}{300R} = (3-x) \times \frac{5}{400R}$ or $(1+x) = \frac{(3-x)5}{4}$

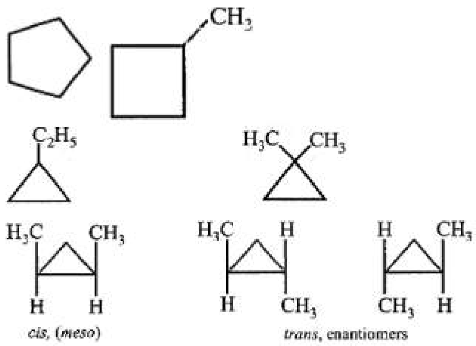
or $4(1 + x) = 15 - 5x$ or $4 + 4x = 15 - 5x$ or $x = \frac{11}{9}$

Hence, new volume of A i.e., $(1 + x)$ will comes as $1 + \frac{11}{9} = \frac{20}{9}$ or 2.22

29. 7

Explanation:

The seven possible cyclic structural and stereoisomers are



30.

(c) A - III, B - I, C - II, D - IV

Explanation:

Medicine	**Action**
Antipyretic	Reduces fever
Analgesic	Reduces pain
Tranquilizer	Reduces stress
Antacid	Reduces acidity (stomach)

31.

(b) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Explanation:

(A) - (III), (B) - (IV), (C) - (I), (D) - (II)

32. **(a)** A - III, B - I, C - IV, D - II

Explanation:

A - III, B - I, C - IV, D - II

Maths

33.

(b) 96

Explanation:

Arrange the letters of the word COCHIN as in the order of dictionary CCHINO.

Consider the words starting from C

There are 5! such words. Number of words with the two C's occupying first and second place = 4!.

Number of words starting with CH, CI, CN is 4! each.

Similarly, number of words before the first word starting with CO = 4! + 4! + 4! + 4! = 96.

The word starting with CO found first in the dictionary is COCHIN. There are 96 words before COCHIN.

34. **(a)** $\left[-\frac{1}{3}, \frac{5}{9}\right]$

Explanation:

Since events A and B are mutually exclusive

$\therefore$ P(A) + P(B) = 1

$\Rightarrow \frac{3x+1}{3} + \frac{1-x}{4} = 1$

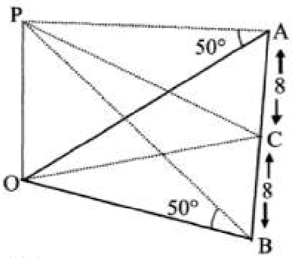
$\Rightarrow$ 12x + 4 + 3 - 3x = 12 $\Rightarrow$ 9x = 5 $\Rightarrow$ x = $\frac{5}{9}$

$\therefore x \in \left[-\frac{1}{3}, \frac{5}{9}\right]$

35. **(a)** $\frac{32}{\sqrt{3}}(2-\sqrt{3})$

Explanation:

In rt. $\triangle$POA



$\frac{\mathrm{OP}}{\mathrm{OA}} = \tan 15^{\circ} \Rightarrow \mathrm{OA} = \mathrm{OP} \cot 15^{\circ}$

In rt. $\triangle$POC

$\frac{\mathrm{OP}}{\mathrm{OC}} = \tan 45^{\circ} \Rightarrow \mathrm{OP} = \mathrm{OC}$

Now, $\mathrm{OP} = \sqrt{\mathrm{OA}^2 - 8^2}$

$\Rightarrow \mathrm{OP}^2 = (\mathrm{OP})^2 \cot^2 15^{\circ} - 64$

$\Rightarrow \mathrm{OP}^2 = \frac{32}{\sqrt{3}}(2-\sqrt{3})$

36.

(b) 3

Explanation:

Given, $y = (c_1 + c_2) \cos (x + c_3) - c_4 e^{x+c_5}$ ......(i)

$\Rightarrow y = (c_1 + c_2) \cos (x + c_3) - c_4 e^{x+c_5}$

Now, let $c_1 + c_2 = A$, $c_3 = B$, $c_4 e^{c_5} = c$

$\Rightarrow y = A \cos (x + B) - ce^x$ ......(ii)

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -A \sin (A + B) - ce^x$ .....(iii)

Again, on differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = -A \cos (x + B) - ce^x$ .....(iv)

$\Rightarrow \frac{d^2y}{dx^2} = -y - 2ce^x$ ......(v)

$\Rightarrow \frac{d^2y}{dx^2} + y = -2ce^x$

Again, on differentiating w.r.t. x, we get

$\frac{d^3y}{dx^3} + \frac{dy}{dx} = -2ce^x$ ...(vi)

$\Rightarrow \frac{d^3y}{dx^2} + \frac{dy}{dx} = \frac{d^2y}{dx^2} + y$ [from Eq.(v)]

Which is a differential equation of order 3.

37. **(b)** $T_{30} = 3454$

(c) $\sum_{k=1}^{20} T_k = 10510$

Explanation: Given, $a_1 - 7$, $d = 8$

Hence, $a_n = 7 + (n - 1) 8$ and $T_1 = 3$

Also $T_{n+1} = T_n + a_n$

$T_n = T_{n-1} + a_{n-1}$...

$T_2 = T_1 + a_1$

$\therefore T_{n+1} = (T_{n-1} + a_{n-1}) + a_n$

$= T_{n-2} + a_{n-2} + a_{n-1} + a_n$...

$\Rightarrow T_{n+1} = T_1 + a_1 + a_2 + ... + a_n$

$\Rightarrow T_{n+1} = T_1 + \frac{n}{2}[2 (7) + (n - 1)8]$

$\Rightarrow T_{n+1} = 3 + n(4n + 3)$ ...(i)

Hence, for $n = 19$; $T_{20} = 3 + (19) (79) = 1504$

For $n = 29$; $T_{30} = 3 + (29) (119) = 3454 \rightarrow (T_{30} = 3454)$

$\sum_{k=1}^{20} T_k = 3 + \sum_{k=2}^{20} T_k = 3 + \sum_{k=1}^{19} (3 + 4n^2 + 3n)$

$= 3 + 3(19) + \frac{3(19)(20)}{2} + \frac{4(19)(20)(39)}{2}$

$= 3 + 10507 = 10510 \rightarrow (\sum_{k=1}^{20} T_k = 10510)$

And similarly $\sum_{k=1}^{30} T_k = 3 + \sum_{k=1}^{29}(4n^2 + 3n + 3) = 35615$

38. **(a)** x - y = 0

(c) x + 7y = 0

Explanation: We know that length of intercept made by a circle on a line is given by $= 2\sqrt{r^2 - p^2}$, where

p = perpendicular distance of the line from the centre of the circle.

Here, circle is $x^2 + y^2 - x + 3y = 0$ with centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$ and radius = $\frac{\sqrt{10}}{2}$

Let L_1: y = mx(any line through origin)

Now, L_2: x + y - 1 = 0 (given line)

ATQ circle makes equal intercepts on L_1 and L_2

$\Rightarrow 2\sqrt{\frac{10}{4} - \frac{\left(\frac{m}{2}+\frac{3}{2}\right)^2}{m^2+1}} = 2\sqrt{\frac{10}{4} - \frac{\left(\frac{1}{2}-\frac{3}{2}-1\right)^2}{2}}$

$\Rightarrow \frac{\left(\frac{m+3}{2}\right)^2}{m^2+1} = 2$

$\Rightarrow m^2 + 6m + 9 = 8m^2 + 8 \Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow (7m + 1)(m - 1) = 0 \Rightarrow m = 1, -\frac{1}{7}$

$\therefore$ The required line L_1 is y = x or $y = -\frac{x}{7}$

i.e., x - y = 0 or x + 7y = 0

39. **(b)** continuous at x = 0

(c) defined for all real x

(d) such that $\frac{dy}{dx} = \frac{1}{3}$ for x < 0

Explanation: Given : x + |y| = 2y

If y < 0 then x - y = 2y

$\Rightarrow y = \frac{x}{3} \Rightarrow x < 0$

If y = 0 then x = 0. If y > 0 then x + y = 2y

$\Rightarrow y = x \Rightarrow x > 0$

$\therefore f(x) = y = \begin{cases} \frac{x}{3}, & x < 0 \\ x, & x \geq 0 \end{cases}$

Continuity at x = 0

$\text{LHL} = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\left(\frac{-h}{3}\right) = 0$

$\text{RHL} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h = 0$

f(0) = 0

$\because$ LHL = RHL = f(0)

$\therefore$ f(x) is continuous at x = 0

Differentiability at x = 0

$Lf' = \frac{1}{3}$; $Rf' = 1$

As Lf' $\neq$ Rf' $\Rightarrow$ f(x) is not differentiable at x = 0

But for x < 0, $\frac{dy}{dx} = \frac{1}{3}$

40. 2.0

Explanation:

Since $e_H = \sqrt{2} \Rightarrow e_E = \frac{1}{\sqrt{2}}$

Since the curves intersect each other orthogonally then the ellipse and the hyperbola are confocal

H : $\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1 \Rightarrow$ foci = (1, 0)

For ellipse $a \cdot e_E = 1 \Rightarrow a = \sqrt{2}$

$(e_E)2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow b^2 = 1$

Length of L.R. $= \frac{2b^2}{a} = \frac{2}{\sqrt{2}} = \sqrt{2}$

41. 16

Explanation:

$$\Delta = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 0 & b-a & e-d \end{pmatrix}$$

$\det A = c(b-a) + d - e$

Given, $a, b, c, d \in \{0, 1\}$

We have, $\det A = 1$ or -1

We take constant of c

Case I: $c = 0, d = 1, e = 0, a, b \in \{0, 1\}$ $d = 0, e = 1, a, b \in \{0, 1\}$

So, total 8 possibilities in this case.

Case II: $c = 1,$

$a = 1, b = 0, d = e = 0$ or $d = e = 1$

$a = 0, b = 1, d = e = 0$ or $d = e = 1$

$a = 0, b = 0, d = 1, e = 0; d = 0, e = 1$

$a = 1, b = 1, d = 1, e = 0, d = 0, e = 1$

Again, a total of 8 possibilities.

So, total number of possible cases $= 16$

42. 9

Explanation:

$\because \vec{r} \times \vec{b} = \vec{c} \times \vec{b} \Rightarrow \vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = \vec{0} \Rightarrow \vec{r} - \vec{c} || \vec{b}$

Let $\vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + \lambda\hat{j}$

$= (1 - \lambda)i + (2 + \lambda)\hat{j} + 3\hat{k}$

$\therefore \vec{r} \cdot \vec{a} = 0 \Rightarrow -1 + \lambda - 3 = 0 \Rightarrow \lambda = 4$

$\therefore \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

Now, $\vec{r} \cdot \vec{b} = 3 + 6 = 9$

43. 0.0

Explanation:

Given, $f(x) = \int\limits_0^{x\tan^{-1}x} \frac{e^{t-\cos t}}{1+t^{2023}} dt$

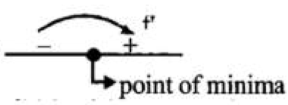
$f'(x) = \frac{e^{x\tan^{-1}x - \cos\left(x\tan^{-1}x\right)}}{1+\left(x\tan^{-1}x\right)^{2023}}\, t\left(\frac{x}{1+x^2} + \tan^{-1}x\right)$

For x < 0, $\tan^{-1}$ x $\in \left(-\frac{\pi}{2}, 0\right)$

For x $\geq$ 0, $\tan^{-1}$ x$\in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow x\tan^{-1}x \geq 0, \forall x \in R$

And $\frac{x}{1+x^2} + \tan^{-1}x = \begin{cases} > 0 & \text{For } x > 0 \\ < 0 & \text{For } x < 0 \\ 0 & \text{Fo } x = 0 \end{cases}$

.i.e 

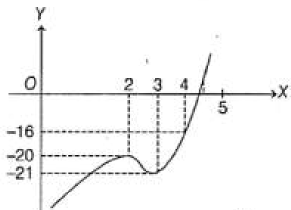
So, f(x) is minimum at x = 0

Here minimum value is f(0) = $\int\limits_0^0 = 0$

44. 7

Explanation:

Given, A = {x|x^2 + 20 $\leq$ 9x} = {x|x $\in$ [4,5]}



Now, f'(x) = 6(x^2 - 5x + 6)

Put f'(x) = 0 $\Rightarrow$ x = 2,3 f(2) = - 20, f(3) = - 21, f(4) = - 16, f(5) = 7

From graph, maximum value of f(x) on set A is f(5) = 7

45. 4

Explanation:

Given, $\bar{z} - z^2 = i\left(\bar{z} + z^2\right)$

It can be written as $\bar{z}$(1 - i) = z^2 (1 + i)

So |$\bar{z}$| |1 - i| = $|z|^2$ |1 + i|

|z| = $|z|^2$ $\Rightarrow$ |z| = 0 or |z| = 1

Let arg(z) = α. So from (i), we get

$2n\pi - \alpha - \frac{\pi}{4} = 2\alpha + \frac{\pi}{4}$

$\Rightarrow \alpha = \frac{1}{3}\left(\frac{4n-1}{2}\right)\pi = \frac{(4n-1)\pi}{6}$

So we will get 3 distinct values of α. Hence there will be total 4 possible values of complex number z.

46.

(c) (I) $\rightarrow$ (Q); (II) $\rightarrow$ (R); (III) $\rightarrow$ (T); (IV) $\rightarrow$(S)

Explanation:

$P(X_i > Y_i)\ P(X_i < Y_i) + P(X_i = Y_i) = 1$

and $P(X_i > Y_i) = P(X_i < Y_i) = p$

For i = 2

$P(X_2 = Y_2) = P(5, 5) + P(4, 4)$

$= \frac{5}{12} \times \frac{5}{12} \times 2 + \frac{1}{6} \times \frac{1}{6}$

$= \frac{25}{72} + \frac{1}{36} = \frac{27}{72} = \frac{3}{8}$

$P(X_2 > Y_2) = P(10, 0) = \frac{5}{12} \times \frac{5}{12} + \frac{5}{12} \times \frac{1}{6} \times 2 = \frac{5}{16}$

$P(X_2 \geq Y_2) = \frac{5}{16} + \frac{3}{8} = \frac{11}{16}$

I $\rightarrow$ Q, II $\rightarrow$ F

For i = 3

$P(X_3 = Y_3) = P(6, 6) + P(7, 7)$

$= \frac{1}{6\times6\times6} + \frac{5}{12} \times \frac{1}{6} \times \frac{5}{12} \times 6 = \frac{77}{432}$

$P(X_3 > Y_3) = \frac{1}{2}\left(1 - \frac{77}{432}\right) = \frac{355}{864}$

III $\rightarrow$ T, IV $\rightarrow$ S

47.

(b) $(P) \rightarrow (2), (Q) \rightarrow (4), (R) \rightarrow (3), (S) \rightarrow (5)$

Explanation:

Given α and β are roots of $x^2 + x - 1 = 0$

$\Rightarrow \alpha + \beta = -1, \alpha\beta = -1$

P. $M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$

Row 1 can be arranged in 3! ways and correspondings the other two rows can be arranged in 2 ways.

$\therefore$ Total number of ways $= 3! \times 2 = 12$

Q. Let $M = \begin{bmatrix} p & m & n \\ m & q & t \\ n & t & r \end{bmatrix}$

Let $m = \alpha, n = \beta, t = 1$

One such arrangement

So m, n, r can be arranged in $3!$ ways and the remaining in 1 way.

$\therefore$ Required total number of ways $= 3! \times 1 = 6$

R. $\begin{bmatrix} 0 & m & n \\ -m & 0 & t \\ -n & -t & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m \\ 0 \\ -t \end{bmatrix}$

We have $D = D_x = D_y = D_z = 0$

Infinite many solutions

S. We have $M = \begin{bmatrix} 1 & \alpha & \beta \\ \beta & \alpha & 1 \\ \alpha & 1 & \beta \end{bmatrix}$

$|M| = \alpha\beta + \alpha^2 + \beta^2 - 1 - \alpha\beta^2 - \alpha^2\beta$

$= \alpha\beta + (\alpha + \beta)^2 - 2\alpha\beta - 1 - \alpha\beta(\alpha + \beta)$

$= (\alpha + \beta)^2 - \alpha\beta - 1 - \alpha\beta(\alpha + \beta)$

$= 1 + 1 - 1 - (-1)(-1) = 1 + 1 - 1 - 1 = 0$

48. **(a)** (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)

Explanation:

Let any point on L_1 is $(2\lambda + 1, -\lambda, \lambda - 3)$ and that on L_2 is $(\mu + 4, \mu - 3, 2\mu - 3)$

For point of intersection of L_1 and L_2

$2\lambda + 1 = \mu + 4, -\lambda = \mu - 3, \lambda - 3 = 2\mu - 3$

$\Rightarrow \lambda = 2, \mu = 1$

$\therefore$ Intersection point of L1 and L_2 is (5, -2, -1)

Equation of plane passing through, (5, -2, -1) and perpendicular to P_1 & P_2 is given by

$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$

$\Rightarrow$ x - 3y - 2z = 13

$\therefore$ a = 1, b = -3, c = -2, d = 13

or (P) $\rightarrow$ (3), (Q) $\rightarrow$ (2), (R) $\rightarrow$ (4), (S) $\rightarrow$ (1)